GSM/D-20   **914**

PARTIAL DIFFERENTIAL EQUATIONS

Paper - BM-232

Time allowed : 3 Hours   *Maximum Marks : 40*

Note : Attempt five questions in all, selecting at least one question from each unit. Question No. 1 is compulsory.

Compulsory Question

1. (i)   Find the differential equation by eliminating the arbitrary constants λ and A from the equation $z = Ae^{-\lambda^2 t}\cos\lambda x$.   1½

    (ii) Classify the differential equation   1½

    $$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

    (iii) Write the one dimensional and two dimensional Heat Equation.   2

    (iv) Examine the compatibility of system of partial equations :   1½

    $p = 3x + 6y$

    $q = 2x - 4y$.

(v) Find the complete integral of partial differential equation p – 3x² = q² – y. 1½

UNIT-I

2. (i) Find out partial differential equation by eliminating arbitrary function from : 4

$$z = \left(\frac{xy}{z}\right)$$

(ii) Solve the partial differential equation : 4

$$p + 3q = z + \cot(y - 3x)$$

3. (i) Find the complete solution of partial differential equation $2xz - px^2 - 2qxy + pq = 0$ by using Charpit's method. 4

(ii) Find the complete integral by Jacobi's method for the equation : 4

$$2x^2y \left(\frac{\partial u}{\partial x}\right)^2 \left(\frac{\partial u}{\partial x}\right) - x^2 \left(\frac{\partial u}{\partial y}\right) - 2y \left(\frac{\partial u}{\partial x}\right)^2 = 0$$

UNIT-II

4. (i) Solve the partial differential equation : 4

$$\frac{\partial^3 z}{\partial x^3} - 3 \frac{\partial^3 z}{\partial x^3 \partial y} + 4 \frac{\partial^3 z}{\partial y^3} = e^{x+2y}$$

(ii) Solve : 4

$$(D^3 - 4D^2D' + 4D'^2D)Z = \cos(2x + y)$$

5. (i) Solve : 4

$$(x^2D^2 - xyDD' - 2y^2D'^2 + xD - 2yD')Z = \log\left(\frac{y}{x}\right) - \frac{1}{2}.$$

(ii) Solve : $(D^2 - 2DD' + D'^2)Z = 12xy$. 4

UNIT-III

6. (i) Classify and reduce the equation :

$$\frac{\partial^2 z}{\partial x^2} - x^2 \frac{\partial^2 z}{\partial y^2} = 0 \quad \text{to canonical form.} \qquad 4$$

(ii) Solve $r + 5s + 6t = 0$. 4

7. (i) Solve $ry^2 + 2xys + x^2t + px + qy = 0$. 4

(ii) Solve $rt - s^2 + 1 = 0$ by Monge's method. 4

UNIT-IV

8. (i) Solve the wave equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$$

by the method of separation of variables. 4

(ii) Solve the Cauchy problem for the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 z}{\partial t^2} = 0, \text{ c} > 0 \text{ subject to the conditions}$$

$$z(x, 0) = f(x) \text{ and } \left[\frac{\partial z}{\partial t}\right]_{t=0} = g(x). \qquad 4$$

9. (i) Find the real characteristics of

$$y \frac{\partial^2 z}{\partial x^2} + (x + y) \frac{\partial^2 z}{\partial x \partial y} + x \frac{\partial^2 z}{\partial y^2} = 0. \qquad 4$$

(ii) Solve : 4

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \,,\, 0 < x < a \text{ and } 0 < y < a$$

subject to the boundary conditions

$4(x, 0) = 4(x, b) = 0$

$4(0, y) = 0$

$4(a, y) = \pi y(b - y).$ 4