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GSE/D-20 **751**

ALGEBRA

Paper–BM-111

Time : Three Hours] [Maximum Marks : 27

Note : Attempt *five* questions in all, selecting *one* question from each section. Question No. 1 is compulsory.

Compulsory Question

1. (a) If A is Hermitian, then show that iA is Skew-Hermitian. 1

(b) Prove that a set which contains the null vector 'o' is linearly dependent. 1

(c) Define Minimal polynomial and minimal equation of a matrix. 1

(d) Multiply the roots of the equation $3x^4 - 2x^2 + 5 = 0$ by 2. 1

(e) Find the least possible number of imaginary roots of the equation $x^9 - x^5 + x^4 + x^2 + 1 = 0$. 1

SECTION–I

2. (a) Show that every square matrix is uniquely expressible as the sum of a Hermitian and a Skew-Hermitian matrix. 3

(b) Express $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ as a product of elementry matrix. 2½

3. (a) Find k if the vectors (2, 0, k), (3, –1, 5) and (5, –1, 1) are linearly dependent. 3

(b) Using the characteristic equation of $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, find A^{-1}. 2½

SECTION–II

4. (a) Show that the system of equations

$$2x - y + z = 4$$

$$3x - y + z = 6$$

$$4x - y + 2z = 7$$

$$-x + y - z = 9$$

is inconsistent. 2½

(b) Reduce $2x^2 + 2y^2 + 3z^2 - 4yz - 4xz + 2xy$ to the canonical form. Also, find the rank and index of the quadratic form. 3

5. (a) Prove that inverse and transpose of an orthogonal matrix are orthogonal. 2½

(b) Prove that $9x^2 + y^2 + 4z^2 + 6xy - 12xz - 4yz$ is positive semi-definite. 3

SECTION–III

6. (a) Use the method of synthetic division to express $2x^5 - x^3 + 10x - 8$ as a polynomial in powers of $(x - 2)$. 3

(b) Solve for x, y, z

$$x + y + z = 6$$

$$xy + yz + zx = 11$$

$$xyz = 6.$$

2½

7. (a) Reduce the cubic $2x^3 - 3x^2 + 6x - 1 = 0$ to the form $Z^3 + 3HZ + G = 0$, where H and G are integers. 3

(b) Find the equation of squared differences of the roots of the cubic $x^3 - 9x^2 + 23x - 15 = 0$. 2½

SECTION–IV

8. (a) Solve the equation $x^3 + 6x^2 + 9x + 4 = 0$ by Cardan's method. 3

(b) Solve the equation $x^4 - 6x^3 + 8x^2 + 2x - 1 = 0$ by Descarte's method. 2½

9. (a) Solve the equation

$x^4 - 4x^3 - 4x^2 - 24x + 15 = 0$ by Ferrari's method. 3

(b) Show that the equation $2x^7 + 3x^4 + 3x + k = 0$ has atleast 4 imaginary roots for all values of k (constant). 2½
