Roll No. ..... **Total Pages : 04**

MDQ/D-20 **5015**

ANALYTICAL MECHANICS AND CALCULUS OF VARIATIONS

MM-502

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section and the compulsory question.

Section I

1. (a) Prove that if the functional : **8**

$$\mathrm{I}\left[y(x)\right]=\int_{x_0}^{x_1}\mathrm{F}\left[x,\, y(x),\, y'(x)\right]dx$$

has an extremum $y(x)$, then $y(x)$ satisfies the differential equation :

$$\frac{\partial \mathrm{F}}{\partial y}-\frac{d}{dx}\left[\frac{\partial \mathrm{F}}{\partial y'}\right]=0$$

(b) Find the external of the functional : **8**

$$\mathrm{I}=\int_1^2\frac{\sqrt{1+y'^2}}{x}dx,\; y(1)=0,\; y(2)=1$$

2. (a) Find the curve joining two points (0, 0, 0) and (1, 2, 4). Also find the distance between these two points. **8**

(b) Determine the equation of Geodesic on the surface of a right circular cylinder of radius '*a*'. **8**

Section II

3. (a) Define and discuss constrained systems and explain their classification. **8**

(b) Derive the Lagrange equations of the second kind. **8**

4. (a) Prove that : **8**

$$T = T_2 + T_1 + T_0$$

(b) Find the equation : **8**

$$Q_i = \frac{\partial \pi_i}{\partial t} + \sum_{k=1}^{n} \left(\frac{\partial \pi_i}{\partial q_k} - \frac{\partial \pi_k}{\partial q_i} \right) \dot{q}_k - \frac{\partial \pi}{\partial q_i}$$

Section III

5. (a) (i) Prove that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$ for a generalised conservative system.

(ii) For the Poisson's bracket, show that : **8**

$$\frac{\partial}{\partial t}(\phi\psi) = \frac{\partial \phi}{\partial t}\psi + \phi\frac{\partial \psi}{\partial t}$$

(b) If f and g are integrals of equation of motion, then (fg) is also an integral of these equations. **8**

6. (a) Show that : **8**

$$I = \int \left[\sum_{i=1}^{n} P_i \delta q_i - H\delta t \right] d\alpha$$

(b) State and prove principle of Least Action. **8**

Section IV

7. (a) Obtain the Hamilton-Jacobi equation : **8**

$$\frac{\partial S}{\partial t} + H\left(t, q_i, \frac{\partial S}{\partial q_i}\right) = 0$$

(b) Show that the transformation : **8**

$$\bar{q}_i = p_i \tan t, \ \bar{p}_i = q_i \cot t, \left(i = 1, \ldots\ldots, n\right)$$

is canonical with :

$$\bar{H} = -H + \frac{1}{\sin t \cos t} \sum_{i=1}^{n} \bar{q}_i \bar{p}_i$$

8. (a) Prove that :

$$(\phi\psi) = -(\bar{\phi}\bar{\psi})$$

that is the Poisson's bracket are invariant under univalent canonical transformations. **8**

(b) Discuss the method of separation of variables. **8**

Compulsory question

9. (i) What are ideal constraints ? **2**

(ii) Obtain the equation : **2**

$$\sum_{i=1}^{n}\left(f_i - m_i\omega_i\right)\delta r_i = 0$$

(iii) Write canonical equations of Hamilton. **2**

(iv) Prove that : **2**

$$\left(\phi\psi\right) = -\left(\psi\phi\right)$$

(v) Define canonical transformations. **2**

(vi) Find the stationary function for the functional : **2**

$$\mathrm{F} = \int_{x_0}^{x_1}\sqrt{1+y'^2}.dx$$

(vii) Define Geodesic. **2**

(viii) What are transition conditions ? **2**